VARIATIONAL PROBLEM OF OPTIMUM HEAT REMOVAL IN RADIAL-TYPE UNITS

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The problem of organizing radial heat removal in a radial-type unit is solved by using a linearized model of the continuous longitudinal-transverse filtration of a fluid in an annular bed with an internal heat source.

Process units of the radial type offer a broad range of potential benefits. The efficient scheme employed for pumping the reactant (coolant) through the bed in such units [1] is the main reason for their relatively low hydraulic resistance, good weight–size characteristics, stable heat and mass transfer, and low recharging costs. Lateral feed of the fluid (gas) into the unit makes it possible to alleviate the effect of adverse hydrodynamic phenomena – stagnant zones, choking, and the valve effect – on working processes occurring in the bed.

These advantages have helped expand the range of application of radial-type units to power generation, chemical and heat engineering, and purification.

Of the many types of such units, we choose to study a class of heat-generating units (Fig. 1) in which ejection of fuel from the combustion zone is the main source of contamination of the environment in accidents. This class of units includes nuclear reactors and some chemical reactors.

In designing a unit of the above-described type, we considered the need to depart from the traditional approach of striving for maximum efficiency. Instead, we wanted to concentrate on increasing the amount of energy in the system. In this case, the volume of the "combustion chamber" and, thus, the amount of fuel in it per unit of capacity would be minimal. The bulk form of the fuel eliminates problems connected with its frequent replenishment. Our main goal in designing the heat-releasing unit was to reduce possible losses from ejections of fuel during accidents.

<u>1. Formulation of the Problem.</u> In order to organize heat removal in units of the above type, it is necessary to determine the through sections of the channels, the form of the end surfaces, the rate of flow of the coolant, and its temperature at the outlet of the unit so as to maximize the limiting thermal load for given dimensions of the unit, pressure losses, and spatial characteristics of the field of energy release.

We will solve the problem on the basis of a two-dimensional model of the continuous longitudinal-transverse motion of a coolant [2]. In this model, the streamlines in the heat-releasing bed are represented by a trinomial

$$x^* = x + a(x)(r - R_1) + \frac{b(x)}{R_2 - R_1}(r - R_1)^2.$$

The model equations are accurate to within second-order infinitesimals of a and b. We will maintain the same degree of accuracy in our calculations.

The heat-releasing bed of the unit consists of monodisperse spherical particles; the particles are kernels, covered with a thin protective shell in which energy is generated. Heat exchange between the bed and the fluid flow is described by the criterional equation [3]

$$\mathrm{Nu} = 0.395\,\mathrm{Re}^{0.64}\,\mathrm{Pr}^{0.33}. \tag{1}$$

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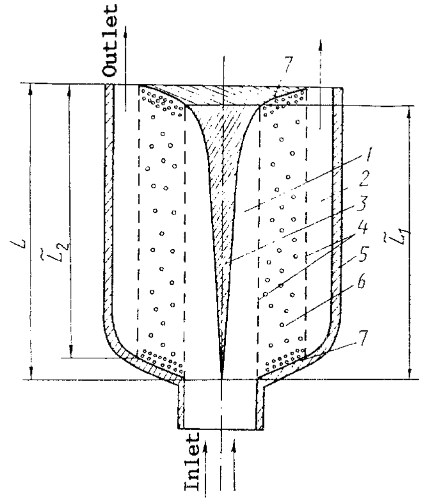

Fig. 1. Basic diagram of a heat-generating process unit of the radial type: 1, 2) distributing and outlet valves; 3) displacement rod; 4) reflecting grates; 5) body (shell); 6) heat-releasing bed; 7) end surfaces.

An excessive increase in the specific energy release leads either to rupture of the shell from thermal stresses or to exceeding of the limiting temperature of the particles [4]. The latter may in turn lead to melting, an intolerable increase in the permeability of the shell and the release of fission fragments, an increase in the chemical activity of the materials of the core and the shell, and several other negative effects. These factors must be eliminated in the organization of heat removal.

We will assume that the flow is quasiisobaric [$(P_{10} - P_{2L})/P_{10} \ll 1$] and that the heat function is given:

$$I = I(T;\ P) \approx I(T;\ P_{10}) \stackrel{\mathrm{def}}{=} \zeta(T),$$

while the thermal conductivity, dynamic viscosity, and Prandtl number obey power relations:

$$\lambda = \lambda_0 (I/I_0)^{\gamma};\quad \mu = \mu_0 (I/I_0)^{\psi};$$

$$c \stackrel{\mathrm{def}}{=} 0.395\,\mathrm{Pr}^{0.33} = c_0 (I/I_0)^{e}. \tag{2}$$

Solving the unidimensional heat-conduction equation for a two-layer sphere with standard boundary conditions [4] with allowance for (1) and (2) and the results in [2], we find the temperature field inside a particle; its maximum value is equal to

$$t = \overline{C} + \overline{B}q + \overline{A}qd^2, \tag{3}$$

where

$$\overline{C} = \zeta^{-1}(I_0 - A - aB - bC);$$

$$\overline{B} = \Omega \left(-\frac{M_1}{2\pi r}\right)^{-0.64} (I_0 - A)^{(0.64\psi - \gamma - e)} \left[1 - \frac{(0.64\psi - \gamma - e)(aB + bC)}{I_0 - A} - \right.$$

$$\left. - a'(r - R_1) - \frac{b'}{R_2 - R_1}(r - R_1)^2\right];$$

$$\Omega = \frac{d^2 \mu_0^{0.64}}{6(1 - \varepsilon)\lambda_0 c_0 I_0^{0.64\psi - \gamma - e} \left[\dfrac{2d}{3(1 - \varepsilon)}\right]^{0.64}};$$

$$\overline{A} = \frac{\dfrac{1}{2\lambda^+} + \dfrac{1}{\lambda^-}(1 - \overline{d})}{12(1 - \varepsilon)\overline{d}};\quad C = \frac{2\pi}{R_2 - R_1}(M_1)^{-1}\int_{R_1}^{r} r(r - R_2)^2 \frac{\partial q}{\partial x}\,dr;$$

$$B = 2\pi (M_1)^{-1}\int_{R_1}^{r} r\frac{\partial q}{\partial x}(r - R_1)\,dr;\ A = 2\pi (M_1)^{-1}\int_{R_1}^{r} rq\,dr;$$

$$M_1 = \frac{2(\pi R)^2 G}{kaF}\left[\frac{3}{2F} - \sqrt{\frac{9}{4F^2} + \frac{ka}{(\pi R)^2}\left(\frac{\xi}{2D} - \frac{1}{F}F'\right)}\right]\Bigg|_1;$$

$$k=\frac{1.7(1-\varepsilon)}{\varepsilon^3 d};\quad \xi_{1,2}=0.145\,(\Delta/D)_{1.2}^{0.322};\ D_1=\frac{2F_1}{\pi\left(R_1+\sqrt{R_1-\frac{F_1}{\pi}}\right)}.$$

Using the equations of the theory of elasticity [5] in an approximation in which the thermal expansion is assumed to be spherically symmetric, we determine the normal components of the stress tensor in the particle. We find that T_{rr} reaches its maximum value at the center (this is the compressive stress), while $T_{\varphi\varphi}$ reaches its maximum value on the outside surface of the shell and leads to its stretching. This quantity is calculated from the formula

$$s=\tilde{C}+\tilde{B}q+\tilde{A}qd^2, \tag{4}$$

where

$$\tilde{C}=\overline{C}K;\quad \tilde{B}=\overline{B}K;\ K=\frac{(1-\overline{d}^3)}{24}W^-+\frac{\overline{d}^3}{24}W^+-\left(\frac{\beta E}{1-\mu}\right)^-;$$

$$\tilde{A}=\frac{1}{288(1-\varepsilon)}\left[W^+d^2\left(\frac{1}{5\lambda^+}+\frac{1-\overline{d}}{\lambda^-}\right)+\frac{W^-}{2\lambda^-}(1+2\overline{d}^3-3\overline{d}^2)\right];$$

$$W^+=\frac{72\beta^+}{\Gamma}\left(\frac{E}{1+\mu}\right)^-\left(\frac{E}{1-2\mu}\right)^+\left(\frac{E}{1-2\mu}\right)^-;$$

$$W^-=\frac{16\beta^-}{\Gamma}\left(\frac{E}{1-\mu}\right)^-\left\{\left(\frac{E}{1-2\mu}\right)^-\left[\left(\frac{E}{1-2\mu}\right)^++2\left(\frac{E}{1+\mu}\right)^-\right]-\right.$$
$$\left.-\overline{d}^3\left(\frac{E}{1+\mu}\right)^-\left[\left(\frac{E}{1-2\mu}\right)^+-\left(\frac{E}{1-2\mu}\right)^-\right]+\frac{\Gamma}{2}\right\};$$

$$\Gamma=\left(\frac{E}{1-2\mu}\right)^-\left[\left(\frac{E}{1-2\mu}\right)^++2\left(\frac{E}{1+\mu}\right)^-\right]+2\overline{d}^3\left(\frac{E}{1+\mu}\right)^-\times$$
$$\times\left[\left(\frac{E}{1-2\mu}\right)^+-\left(\frac{E}{1-2\mu}\right)^-\right].$$

We will assume that, from the strength viewpoint, the bed is capable of functioning if the integrity of the shells covering the particles has not been disturbed. Here, the condition of the particles themselves is unimportant. Then the two conditions which limit the energy release will have the form

$$(t-\overline{T})\stackrel{\text{def}}{=}\varphi\leqslant 0;\quad (s-\overline{S})\stackrel{\text{def}}{=}\alpha\leqslant 0, \tag{5}$$

where t and s are described by Eqs. (3) and (4).

A system of differential equations of heat removal, solved relative to the derivatives, was presented in [2]. Since the right sides of these equations are cumbersome, we will agree that the following quantities are equal:

$$\begin{gathered}Y_1=G_1;\quad Y_2=G_2;\quad y_3=u;\quad y_4=a;\quad y_5=b;\\ y_6=P_1;\quad y_7=P_2;\quad q=\langle q\rangle\,\delta(\mathbf{r});\quad \langle q\rangle=\text{const};\\ y_8=\langle q\rangle;\quad Y_{17}=F_1;\quad Y_{18}=F_2;\quad y_{19}=F_1';\quad y_{20}=F_2'\end{gathered} \tag{6}$$

and we will represent the system in generalized form

$$y_i'=f_i(x;\ y_1;\ \ldots;\ y_{20})\quad (i=\overline{1,\ 8}), \tag{7}$$

formally reasoning that the right sides of Eqs. (7) depend explicitly on all of the unknowns $(y_1;\ \ldots;\ y_{20})$; we adopt the same position in regard to α, φ, Y, and all of the functions, which we will henceforth designate through the letters f, ω, Φ, and θ. According to (6),

$$y_k'=\left[y_{k+2}-\frac{\partial Y_k}{\partial x}-\frac{\partial Y_k}{\partial y_i}f_i\right]\Big/\frac{\partial Y_k}{\partial y_k}=f_k; \tag{8}$$

$$y_8' = f_8 = 0 \quad (i = \overline{1,\ 20};\quad i \neq 17 \wedge 18);\quad k = 17 \wedge 18.$$

Information on $Y(\cdot)$ and the unknowns $y_9;\ \ldots;\ y_{20}$ will be presented in the course of the solution.

From [2] we take boundary conditions that are consistent with the problem formulated above:

$$Y_1|_{x=\tilde{L}_1} = 0;\quad Y_2|_{x=L-\tilde{L}_2} = 0;\quad Y_{17}|_{x=\tilde{L}_1} = 0;\quad Y_{18}|_{x=L-\tilde{L}_2} = 0;$$
$$y_3|_{x=L-\tilde{L}_2} = 0;\quad y_6|_{x=0} = P_{10};\quad y_7|_{x=L} = P_{2L};\quad Y_1|_{x=0} = Y_2|_{x=L}. \tag{9}$$

Let us point out the limitations on the through sections of the channels and the fluid flow rates in them:

$$\forall x \in [0;\ \tilde{L}_1),\quad \Xi_1 \geqslant Y_{17} > 0,\quad Y_1 \geqslant 0,\ Y_1' \leqslant 0;$$
$$\forall x \in (L - \tilde{L}_2;\ L],\quad \Xi_2 \geqslant Y_{18} > 0,\quad Y_2 \geqslant 0,\quad Y_2' \geqslant 0, \tag{10}$$

which follow from the requirements imposed on the dimensions of the unit and the kinetics of coolant motion.

We will assume that we know the quantities $R_{1,2}$, λ_0, μ_0, c_0, I_0, γ, ψ, e, $\zeta(T)$, d, $\bar{d}$, ε, λ^+, λ^-, $(\Delta/D)_{1,2}$, $\Xi_{1,2}$, E^+, E^-, μ^+, μ^-, β^+, β^-, $\bar{T}$, $\bar{S}$, L, P_{10}, P_{2L}, $\delta(r)$, and $\rho = \rho(T)$.

2. Solution of the Heat Removal Problem. Mathematically, this problem amounts to a search for the functions $y_1;\ \ldots;\ y_{20}$ that minimize the functional

$$J = -\int_V y_8 \delta(r)\, dV = \min, \tag{11}$$

in the presence of nonholonomic constraints (7), (8) with boundary conditions (9) and limitations in the form of inequalities (5) and (10). It follows from Eqs. (11) and (5) that the extremal y_8 is positive. Thus, the requirement that the flow rates be monotonic [see (10)] is equivalent to

$$-A - aB - bC \overset{\text{def}}{=} \omega \geqslant 0. \tag{12}$$

This relation will also be used in the solution of the problem.

For nonseparated flow of the coolant (separation zones appreciably lower the cooling rate), the ends of the bed should coincide with the surfaces formed at the edges of the trajectories of a coolant obeying the classical filtration law [2]. Since the velocity field is not known from the conditions of the problem, it is impossible to a priori determine the volume and configuration of the bed. Thus, Eq. (11) turns out to have indeterminate limits of integration:

$$J = -2\pi \int_{R_1}^{R_2} r dr \int_{\bar{a}}^{\bar{b}} y_8 \delta(x;\ r)\, dx = \min;$$
$$\bar{a} = L - [(y_4)_L + (y_5)_L](R_2 - R_1) + (y_4)_L (r - R_1) + (y_5)_L \frac{(r - R_1)^2}{R_2 - R_1};$$
$$\bar{b} = (y_4)_0 (r - R_1) + (y_5)_0 \frac{(r - R_1)^2}{R_2 - R_1}. \tag{13}$$

We use symmetrical unit functions to construct a functional with fixed limits equivalent to (13):

$$J = -2\pi \int_{R_1}^{R_2} \int_0^L \delta y_8 (H_2 - H_1)\, r dr dx = \min, \tag{14}$$

where

$$H_1 = U(\bar{b} - x);\ H_2 = U(\bar{a} - x);$$
$$U(\cdot) = \lim_{m \to 0} \frac{1}{2}\left[1 + \operatorname{erf} \frac{1}{m}(\cdot)\right].$$

Here and below, we use approximate expressions of U which have proven expedient in numerical experiments we have performed.

Using the property of U, we separate (14) into two terms in such a way that each term depends only on the sought variables taken on identical edges:

$$J = \overline{\Phi}_1 + \overline{\Phi}_2,$$

where

$$y_8 \int\limits_{R_1}^{R_2} r dr \int\limits_{\overline{x}}^{[(y_4)_0+(y_5)_0](R_2-R_1)} \delta H_1 dx \overset{\text{def}}{=} \overline{\Phi}_1 (y_8;\ (y_4)_0;\ (y_5)_0;\ \overline{x});\ \overline{x} = 0;\ -y_8 \int\limits_{R_1}^{R_2} \int\limits_0^L \delta H_2 r dr dx \overset{\text{def}}{=} \overline{\Phi}_2 (y_8;\ (y_4)_4;\ (y_5)_L;\ L).$$

Such summation makes it possible to use the Boltz—Meyer transform [6] and remove the unknown parameters $(y_4)_{0,L}$ and $(y_5)_{0,L}$ from under the integral sign. We will indicate the key aspects of this mathematical operation. We construct the functions $\Phi_{1,2}$ with the simple substitution of $(y_4)_0 \wedge (y_4)_L = y_4$, $(y_5)_0 \wedge (y_5)_L = y_5$, $\overline{x} \wedge L = x$ into $\overline{\Phi}_{1,2}$. We determine the total derivatives with respect to x from $\Phi_{1,2}$ and designate them as y_9'; y_{10}':

$$y'_{9,10} = \frac{\partial \Phi_{1,2}}{\partial x} + \frac{\partial \Phi_{1,2}}{\partial y_i} f_i \overset{\text{def}}{=} f_{9,10};\ (i = \overline{4,\ 8};\ i \neq 6 \wedge 7). \tag{15}$$

Then we represent the functional (14) in traditional form

$$J = \int\limits_0^L (f_9 + f_{10})\, dx = \min, \tag{16}$$

and we add (15) to (7) with the conditions

$$\begin{aligned} &(y_9 + \Phi_1 \overset{\text{def}}{=} \overline{N}_1)|_{x=0} = 0;\ \ (y_{10} \overset{\text{def}}{=} \overline{N}_2)|_{x=0} = 0; \\ &(y_9 \overset{\text{def}}{=} \overline{M}_1)|_{x=L} = 0;\ \ (y_{10} - \Phi_2 \overset{\text{def}}{=} \overline{M}_2)|_{x=L} = 0. \end{aligned} \tag{17}$$

Limitations (5) and (12) depend on the two running coordinates x and r, but the sought quantities themselves are functions only of x. This fact allows us, by discretizing Eqs. (5) and (12) with respect to r:

$$r = r_j = R_1 - j\,\frac{(R_2 - R_1)}{M};\ \alpha_j = \alpha(r_j);\ \varphi_j = \varphi(r_j);\ \omega_j = \omega(r_j),$$

and using them as a basis for introducing the penalty function h, to change over from (5), (7), (8), (10), (12), and (15)-(16) to a variational problem with one independent variable x. We can then proceed to relations (7), (8), (10), and (15) and the functional

$$J = \int\limits_0^L (f_9 + f_{10})\, h dx = \min, \tag{18}$$

where

$$h = \prod_{j=1}^{M} U(-\alpha_j)\, U(-\varphi_j)\, U(\omega_j);\quad U(\cdot) = \lim_{m\to 0}\left[\frac{1}{2} + \frac{1}{\pi}\operatorname{arctg}\frac{(\cdot)}{m}\right].$$

We will construct the relations $G_1 = Y_1$, $G_2 = Y_2$, $F_1 = Y_{17}$, $F_2 = Y_{18}$ with an infinite domain and a finite set of values of the functions

$$G_{1,2} = Y_{1,2} = \lim_{m\to 0}\left\{\exp y_{1,2} \pm m\left(\frac{1}{2} - \frac{x}{L}\right) + \overline{k}_{1,2} y_{1,2} U_+ [\pm(x - x_{L,0})]\right\}; \tag{19}$$

$$F_{1,2} = Y_{17,18} = \lim_{m\to 0}\left\{\frac{\Xi_{1,2}}{\pi}\left(\operatorname{arctg} y_{17,18} + \frac{\pi}{2}\right) \pm m\left(\frac{1}{2} - \frac{x}{L}\right) + \overline{k}_{17,18} y_{17,18} U_+ [\pm(x - x_{L,0})]\right\},$$

where

$$U_+(\cdot) = 1 - \lim_{m\to 0} 2^{-(\cdot)^2 \exp\left[\frac{1}{m}(\cdot)\right]};$$

$$x_L = \tilde{L}_1 = L - [(y_4)_L + (y_5)_L](R_2 - R_1);$$

$$x_0 = L - \tilde{L}_2 = [(y_4)_0 + (y_5)_0](R_2 - R_1).$$

Inequalities (10) are automatically accounted for with their inclusion in the mathematical expressions. Let us explain the functional role of each term in (19). The first terms keep the extremals $F_{1,2}$ and $G_{1,2}$ within the necessary interval. The second terms ensure the existence of finite values of y_1, y_2; y_{17}, y_{18} at the points x_0, x_L. Due to design features of the unit, the through sections of the channels and, thus, the coolant flow rates are zero at these points. The third terms remove the limitations of the functions F_1, G_2; F_2, G_2 at $x > x_L$, $x < x_0$.

We bring boundary conditions (9) into conformity with the limits of integration (0; L) by resorting to linear extrapolation

$$\left.\left[Y_k - \left(\frac{\partial Y_k}{\partial x} + \frac{\partial Y_k}{\partial y_k} f_k\right)(y_4 + y_5)(R_2 - R_1) \overset{\text{def}}{=} \overline{M}_{3,4}\right]\right|_{x=L} = 0;\ (k = 1 \wedge 17);$$

$$\left.\left[Y_n + \left(\frac{\partial Y_n}{\partial x} + \frac{\partial Y_n}{\partial y_n} f_n\right)(y_4 + y_5)(R_2 - R_1) \overset{\text{def}}{=} \overline{N}_{3,4}\right]\right|_{x=0} = 0;\ (n = 2 \wedge 18); \tag{20}$$

$$[y_3 + f_3(y_4 + y_5)(R_2 - R_1) \overset{\text{def}}{=} \overline{N}_5]|_{x=0} = 0;$$

$$(y_6 - P_{10} \overset{\text{def}}{=} \overline{N}_6)|_{x=0} = 0;\ (y_7 - P_{2L} \overset{\text{def}}{=} \overline{M}_5)|_{x=L} = 0.$$

Then the equality of the coolant flow rates at the inlet and outlet of the unit will be expressed in the form

$$\int_0^L \left[\frac{\partial}{\partial x}(Y_1 + Y_2) + \frac{\partial Y_1}{\partial y_1} f_1 + \frac{\partial Y_2}{\partial y_2} f_2\right] dx - \varkappa_1|_{x=L} - \varkappa_2|_{x=0} = 0, \tag{21}$$

where

$$\varkappa_{1,2} = \left(\frac{\partial Y_{1,2}}{\partial x} + \frac{\partial Y_{1,2}}{\partial y_{1,2}} f_{1,2}\right)(y_4 + y_5)(R_2 - R_1).$$

The Boltz–Meyer transformation makes it possible to represent (21) as a set of additional nonholonomic constraints:

$$y'_{11,12} = \frac{\partial \varkappa_{1,2}}{\partial x} + \frac{\partial \varkappa_{1,2}}{\partial y_i} f_i \overset{\text{def}}{=} f_{11,12};\ i = \overline{1,\ 20};\ i \neq 11 \wedge 12; \tag{22}$$

together with the isoperimetric constraints

$$\int_0^L \left\{\left[f_{11} + f_{12} + \frac{\partial}{\partial x}(Y_1 + Y_2) + \frac{\partial Y_1}{\partial y_1} + \frac{\partial Y_2}{\partial y_2} f_2\right] \overset{\text{def}}{=} \Theta_1\right\} dx = 0 \tag{23}$$

and the boundary conditions

$$(y_{11} \overset{\text{def}}{=} \overline{N}_7)|_{x=0} = 0;\ (y_{11} + \varkappa_1 \overset{\text{def}}{=} \overline{M}_6)|_{x=L} = 0;$$

$$(y_{12} - \varkappa_2 \overset{\text{def}}{=} \overline{N}_8)|_{x=0} = 0;\ (y_{12} \overset{\text{def}}{=} \overline{M}_7)|_{x=L} = 0. \tag{24}$$

If we use (7), (15), (18), (19), (22), and (23) to construct the auxiliary functional needed to solve the variational problem by the Lagrange method, we find that the integrand contains the sought quantities in the form of the constants $x_{0,L}$. Thus, we designate them as $y_{13,14}$ in (19) and consider them to be functions of x. Also, so as not to change the meaning of the problem, we impose two constraints

$$y'_{13,14} \stackrel{\text{def}}{=} f_{13,14} = 0;$$

the equalities between the unknowns on the edges

$$y_{13}|_{x=L} - [x - (R_2 - R_1)(y_4 + y_5)]|_{x=0} = 0;$$
$$y_{14}|_{x=0} - [x - (R_2 - R_1)(y_4 + y_5)]|_{x=L} = 0,$$

which would prevent us from finding suitable boundary conditions, are replaced by the equivalent relations

$$\int_0^L \theta_2 dx = 0; \int_0^L \theta_3 dx = 0; \theta_2 = f_{15}; \theta_3 = f_{16};$$

$$y'_{15} = y'_{16} = 1 - (R_2 - R_1)(f_4 + f_5) \stackrel{\text{def}}{=} f_{15} \stackrel{\text{def}}{=} f_{16}; \tag{25}$$

$$[y_{15} - x - (R_2 - R_1)(y_4 + y_5) \stackrel{\text{def}}{=} \overline{N}_9]|_{x=0} = 0; (y_{15} - y_{13} \stackrel{\text{def}}{=} \overline{M}_8)|_{x=L} = 0;$$
$$(y_{16} - y_{14} \stackrel{\text{def}}{=} \overline{N}_{10})|_{x=0} = 0; [y_{16} - x - (R_2 - R_1)(y_4 + y_5) \stackrel{\text{def}}{=} \overline{M}_9]|_{x=L} = 0.$$

In the given case, the auxiliary functional will have the form

$$\int_0^L [\vartheta + C_n\theta_n + \Lambda_i (y'_i - f'_i]\, dx = \min; \; i = \overline{1,\ 18}; \; n = \overline{1,\ 3}, \tag{26}$$

where

$$\vartheta = (f_9 + f_{10})\, h.$$

Since the Eulerian system corresponding to it

$$\frac{\partial \vartheta}{\partial y_k} + C_n \frac{\partial \theta_n}{\partial y_k} - \Lambda_i \frac{\partial f_i}{\partial y_k} - \Lambda'_k = 0; \; i = \overline{1,\ 18}; \; k = \overline{1,\ 18}; \tag{27}$$

$$\frac{\partial \vartheta}{\partial y_l} + C_n \frac{\partial \theta_n}{\partial y_l} - \Lambda_i \frac{\partial f_i}{\partial y_l} \stackrel{\text{def}}{=} \overline{M}_{10,11} \stackrel{\text{def}}{=} \overline{N}_{11,12} = 0; \; l = \overline{19,\ 20}; \tag{28}$$

$$y'_i = f_i \tag{29}$$

consists of differential and transcendental equations, and since the boundary values are unknown, we replace Eqs. (28) in the system by their derivatives with respect to x, solved relative to $y_{19,20}'$:

$$y'_{20} = \frac{\varepsilon_{20}\sigma_{19} - \varepsilon_{19}N}{N^2 - \sigma_{19}\sigma_{20}} \stackrel{\text{def}}{=} f_{20}; \quad y'_{19} = \frac{\varepsilon_{19}\sigma_{20} - \varepsilon_{20}N}{N^2 - \sigma_{19}\sigma_{20}} \stackrel{\text{def}}{=} f_{19},$$

where

$$\sigma_l = \frac{\partial^2 \vartheta}{\partial y_l^2} + C_n \frac{\partial^2 \theta_n}{\partial y_l^2} - \Lambda_i \frac{\partial^2 f_i}{\partial y_l^2};$$

$$\varepsilon_l = \frac{\partial^2 \vartheta}{\partial x \partial y_l} + C_n \frac{\partial^2 \theta_n}{\partial x \partial y_l} + f_k \left(\frac{\partial^2 \vartheta}{\partial y_k \partial y_l} + C_n \frac{\partial^2 \theta_n}{\partial y_k \partial y_l} \right) -$$

$$- \frac{\partial f_i}{\partial y_l} \left(\frac{\partial \vartheta}{\partial y_i} - \Lambda_k \frac{\partial f_k}{\partial y_i} + C_n \frac{\partial \theta_n}{\partial y_i} \right) - \Lambda_i \left[\frac{\partial^2 f_i}{\partial x \partial y_l} + \frac{\partial^2 f_i}{\partial y_k \partial y_l} f_k \right]; \tag{30}$$

$$N = \frac{\partial^2 \vartheta}{\partial y_{19} \partial y_{20}} + C_n \frac{\partial^2 \theta_n}{\partial y_{19} \partial y_{20}} - \Lambda_i \frac{\partial^2 f_i}{\partial y_{19} \partial y_{20}},$$

and we insert Eqs. (28) into the general transversality condition

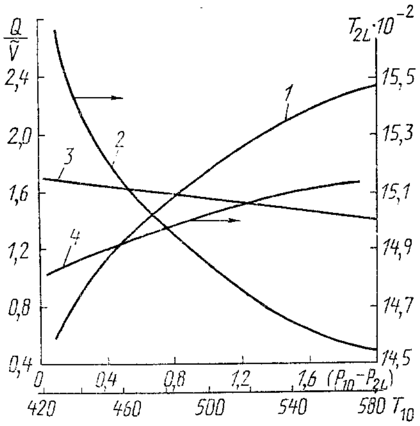

Fig. 2. Dependences of $Q/\tilde{V}$, MW/dm³, and T_{2L}, K, on the pressure drop ($P_{10} - P_{2L}$), MPa, at T_{10} = 473 K (1, 2); on gas temperature at the inlet of the unit, K, with P_{2L} = 9.2 MPa (3, 4).

$$\begin{cases} \Lambda_i + \sum\limits_{\nu=1}^{11} \tau_\nu \left.\dfrac{\partial \overline{M}_\nu}{\partial y_i}\right|_{x=L} = 0, \\ \boldsymbol{M}_\nu|_{x=L} = 0; \end{cases}$$
$$\begin{cases} \Lambda_i + \sum\limits_{l=1}^{12} \chi_l \left.\dfrac{\partial \overline{N}_l}{\partial y_i}\right|_{x=0} = 0, \\ \overline{N}_l|_{x=0} = 0. \end{cases} \tag{31}$$

Closed system of differential equations (27), (29), and (3) with boundary conditions (31) and isoperimetric conditions (23) and (25) (passage to the limit $m \to 0$ during the solution of this system is accomplished by means of spline interpolation [7]) describes the optimum geometry of the structural elements of the unit and optimum heat-removal parameters; the presence of sufficient conditions for the minimum of functional (26) is checked by means of the test proposed in [8]. (The problem of optimizing heat removal with the condition of attainment of the maximum value of ηQ is solved by a similar method.)

3. Numerical Example. We will use the mathematical optimization model to determine the dependence of $Q/\tilde{V}$ and T_{2L} on the pressure drop in the unit and the gas temperature T_{10}. The need for this information arises in the calculation of the thermodynamic cycle of the unit. The initial data: R_1 = 0.21 m, R_2 = 0.37 m, L = 1.0 m, Ξ_1 = 0.1385 m², Ξ_2 = 0.194 m²; d = $1.8\cdot10^{-3}$ m, $\bar{d}$ = 0.8; ε = 0.4; γ = 0.5; e = 0; ψ = 0.5; c_0 = 0.395; $\bar{k}_{1,2} = 10^{-7}$, $\bar{k}_{17} = 10^{-6}$, $\bar{k}_{18} = 10^{-4}$; M = 20; $(\Delta/D)_{1,2}$ = 0.05; $\zeta(T) = c_pT$; coolant – CO_2 (its properties were given in the handbook [9]); P_{10} = 10 MPa; $\lambda^+ = \lambda^-$ = 21 W/(m·K); $E^+ = 0.15\cdot10^{12}$ N/m²; $E^- = 0.2\cdot10^{12}$ N/m²; μ^+ = 0.24; μ^- = 0.27; $\beta^+ = 0.150\cdot10^{-4}$ 1/K; $\beta^- = 0.178\cdot10^{-4}$ 1/K; $\bar{T}$ = 1700 K; $\bar{S} = 0.44\cdot10^9$ N/m²; $\delta(\mathbf{r}) = \left(\frac{1}{\pi} + \frac{\pi-1}{2}\sin\frac{\pi x}{L}\right)\left[\frac{3R_2 - r}{3.2(R_2 - R_1)}\right]$. The pressure P_{2L} and temperature T_{10} were varied within the ranges P_{2L} = [9.9; 8.0] MPa, T_{10} = [423; 573] K.

The results of the numerical experiment are shown in Fig. 2. It should be noted that an increase in the pressure drop in the unit is accompanied by an increase in the limitingly allowable energy release, with a tendency to reach the saturation line with a value of 2.37 MW/dm³; at the same time, the optimum temperature T_{2L} decreases and approaches the lower boundary of 1450 K. Such behavior of curves 1 and 2 is caused by a choking effect [10]. The graphs of the functions $Q/\tilde{V}$ and T_{2L} with the argument T_{10} are approximately straight lines; the first quantity decreases monotonically, while the second increases with a derivative substantially less than unity (with an increase in T_{10} by 150 K, T_{2L} increases by only 30 K).

CONCLUSIONS

The calculations show that the above-mentioned qualitative features of heat removal do not depend on the dimensions of the unit, the properties of the fuel, or the form of the energy-release field; they are intrinsic to nearly all heat-generating units of the radial type. This makes it possible to consider radial-type units as being among the most promising types of process vessels. Their introduction will have a positive effect on safety in power engineering as a whole.

NOTATION

x, r, cylindrical coordinates; R_1 (R_2), radius of the internal (external) lateral surface of the bed; t, temperature at the center of a particle; ρ, λ, μ, cp, I, density, thermal conductivity, dynamic viscosity, heat capacity, and enthalpy of the coolant; q, volumetric energy release in the bed; d, diameter of a particle; $\bar{d}$, relative diameter of the kernel (core) of a particle; $\tilde{V} = \pi L(R_2{}^2 - R_1{}^2)$; G, flow rate of gas in the channel; F, through section of the channel; ξ, drag coefficient; Δ, height of roughness on the channel walls; D, equivalent hydraulic diameter of channel; s, shear stresses on the external surface of the shell of a particle; E^+, E^-, μ^+, μ^-, β^+, β^-, λ^+, λ^-, Young's modulus, Poisson's ratio, coefficient of thermal expansion, and thermal conductivity of the materials of the core and shell of a particle, respectively; P, gas pressure; δ(r), nonnormalized form of the field of energy release in the bed; $\bar{T}$, limitingly allowable particle temperature; $\bar{S}$, maximum possible shear stresses in the shell; L, length of unit; V, volume of bed; T, gas temperature; $\bar{L}$, length of channel; Ξ, upper bound of the through section of the channel; M, degree of discretization of the problem with respect to the radius; η, efficiency of the power plant; u, thermal load of the flow in the outlet channel; $\bar{k}_{1,2}$, $\bar{k}_{17,18}$, scale factors; Λ, τ, χ, Lagrangian multipliers. Indices: 1, parameters pertaining to the distributing channel; 2, parameters pertaining to the outlet channel; quantities at the inlet and outlet of the unit are denoted by 0 and the letter L, respectively; relations valid for both the distributing and outlet channels have subscripts 1, 2.

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